Section 8.3

Substitution Just Works!

Our use of substitution has amounted thus far to a convenient change in variables.

$$\int f(u(x))u'(x)dx = \int f(u)du$$

or written much less familiarly

$$\int f(x)\,dx; \quad u = u(x) \quad \Longrightarrow \quad \int f(x)\,dx = \int f(u^{-1}(x))\frac{1}{u'(u^{-1}(x))}dx.$$

The second form suggests that we can substitute any function we like into f!

$$\int f(x)\,dx; \quad x = g(y) \quad \Longrightarrow \quad \int f(x)\,dx = \int f(g(y))\frac{1}{g'(y)}dy$$

with the challenge being to find a useful $g(y)$.

Example 8.3.1. Compute the definite integral $\int_0^r \sqrt{r^2 - x^2}dx$.

Example 8.3.2. Evaluate the following integrals.

1. $\displaystyle\int_0^{3/2} \frac{dx}{(9-x^2)^{3/2}}$

2. $\displaystyle\int_0^{2} \frac{x^2}{x^2+4}dx$

3. $\displaystyle\int \frac{dx}{\sqrt{x^2+2}}dx$

4. $\displaystyle\int_{\sqrt{2}}^{2} \frac{\sqrt{x^2-1}}{x}dx$

5. $\displaystyle\int \frac{dx}{\sqrt{3-2x-x^2}}$

6. $\displaystyle\int \frac{dy}{y^2+6y+18}$